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# Comparison of Next-to-Leading Order Calculations for Jet Cross Sections in Deep-Inelastic Scattering

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**Abstract:** We compare different next-to-leading order calculations of jet cross sections in deep-inelastic scattering as implemented in the programs DISASTER++, DISENT, JETVIP and MEPJET. In all phase space regions under study DISENT and DISASTER++ agree better than 2%. MEPJET shows systematic deviations of being typically 5–8% lower than the other programs. The JETVIP results show a significant dependence on the phase space slicing parameter  $y_{\text{cut}}$ . In the cases where the  $y_{\text{cut}}$  dependence within  $10^{-4} \leq y_{\text{cut}} \leq 10^{-3}$  is smaller than 3% the JETVIP results are often comparable with the DISENT and DISASTER++ results.

## 1 Introduction

At present four different Monte Carlo programs are available for the computation of jet quantities in deep-inelastic scattering (DIS) to next-to-leading order (NLO) in the strong coupling constant  $\alpha_s$ : MEPJET [1, 2], DISENT [3], DISASTER++ [4] and JETVIP [5]. Since all of these claim to be exact calculations, they should produce identical results (within numerical precision). In this contribution we compare the leading-order (LO) and the NLO predictions of these programs to test whether they are compatible. The comparisons are performed in typical phase space regions where HERA analyses are currently made.

## 2 Program Overview

The four programs allow to calculate next-to-leading order parton cross sections with arbitrary cuts. They differ in the techniques used and in several details. A short overview on the four programs is given in table 1. For a detailed discussion of the single topics we refer to the program manuals.

The programs can be classified by the method that is applied to cancel the collinear and infrared singularities. Two general methods are available, the phase space slicing method and the subtraction method. The phase space slicing method employs a technical cutoff ( $s_{\text{min}}$  or

	MEPJET[1]	DISENT[3]	DISASTER++[4]	JETVIP[5]
version	2.2	0.1	1.0.1	1.1
method	PS slicing	subtraction	subtraction	PS slicing
1+1,2+1	NLO	NLO	NLO	NLO
3+1	LO	LO	LO	LO
4+1	LO	—	—	—
full event record	✓	✓	✓	(✓)
scales	all	factorization: $Q^2$ , fixed renormalization: all	all	all
flavour dependence	switch	switch	full	switch
quark masses				
in LO x-section	LO	—	—	—
resolved $\gamma$ contribution				
in LO/NLO x-section	—	—	—	NLO
electroweak contribution				
in LO/NLO x-section	LO	—	—	—
polarized x-section	NLO	—	—	—

Table 1: *Comparison of the different features of the programs. Note that DISENT has been changed with respect to the official version to implement e.g. the running electromagnetic coupling constant. The common NLO library [6] version 0.2 has been used to interface DISASTER++.*

$y_{\text{cut}}$ ). Correct results are only obtained for sufficiently small values of this parameter. The cutoff independence has to be checked for every investigated observable/scenario. In practice this test is performed by comparing multiple runs with different (small) cutoff parameters. The subtraction method does not apply such a cutoff.

All programs are able to calculate single jet and dijet observables in next-to-leading order, i.e.  $\mathcal{O}(\alpha_s^1)$  or  $\mathcal{O}(\alpha_s^2)$  for processes with one or two partons in the Born process. Processes with a higher number of particles in the Born graph are available in leading order only.

In order to apply arbitrary cuts on the final state, the full event record of all incoming and outgoing particles is needed. The full event record is available for all programs with the exception of the azimuthal angle  $\phi$  of the scattered electron wrt. the outgoing partons in the JETVIP program. In JETVIP the  $\phi$  dependence of the matrix elements is integrated analytically. Since this angle is not available, the full vector of the Lorentz boost from the Breit frame to the HERA laboratory frame can only be calculated under the assumption of a flat distribution in  $\phi$ . At larger  $Q^2$  this can lead to an error of at maximum 5-7% when angular jet cuts in the HERA laboratory frame are applied [2]. Therefore no such cuts are used in our test scenarios.

In perturbative QCD calculations two scales are introduced: the renormalization ( $\mu_r$ ) and the factorization scale ( $\mu_f$ ). All programs allow to identify the renormalization scale with arbitrary variables, e.g. proportional to kinematic variables ( $Q$ ) or to final state quantities ( $E_{T,\text{jet}}$ ). The same is true for the factorization scale, except for DISENT. In DISENT the factorization scale is restricted to variables that are independent of the hadronic final state, i.e. proportional to kinematic variables ( $Q$ ) or to constant values. To keep the checks simple, we stick to the choice of  $\mu = Q$  for both scales.

At very low and at very high  $Q^2$ , effects changing the cross section become more and more important. At high  $Q^2$  the exchange of  $Z$  and  $W$  bosons can not be neglected while at low  $Q^2$  the contributions from resolved photons to jet cross sections become sizable. In other regions of phase space effects from quark masses can also become relevant. Since these different effects can only be calculated by single programs (see table) they have not been considered in the present comparison.

## 3 Comparison of the Results

### 3.1 Technical Settings

For all NLO calculations as well as for the LO calculations we are using renormalization and factorization scales  $\mu_r = \mu_f = Q$  and the 2-loop formula for the running of  $\alpha_s$  (taken from PDFLIB [7]). Throughout we are using CTEQ4M parton density functions [8] (taken from PDFLIB). All cross sections are calculated for a running electromagnetic coupling constant<sup>1</sup> and are performed in the  $\overline{\text{MS}}$  scheme with five active flavors.

### 3.2 Scenarios for the Comparisons

At NLO the jet cross sections depend on the jet definition and the recombination scheme. For all comparisons we are using the inclusive  $k_\perp$  algorithm [9] in the Breit frame. It has been shown that this jet definition is infrared safe to all orders [10] and less affected by hadronization corrections than other jet definitions [11]. Particles are recombined in the  $E_T$ -scheme [12] in which the jet  $E_T$  is obtained from the scalar sum of the particle  $E_T$ , the pseudorapidity and the azimuth angle are calculated as  $E_T$ -weighted averages from the particle quantities. In all cases we calculate *inclusive* jet cross sections (i.e. cross sections for the production of events with at least two jets that pass the jet cuts). The jets are indexed in descending order in their transverse energies in the Breit frame ( $E_{T1} \geq E_{T2}$ ).

The  $ep$  center of mass energy squared is set to  $s = 4 \cdot 27.5 \cdot 820 \text{ GeV}^2 = 90200 \text{ GeV}^2$  (corresponding to the HERA running conditions in 1994-97). What will later be called the “central scenario” is defined as follows:

$$30 < Q^2 < 40 \text{ GeV}^2, \quad 0.2 < y < 0.6, \quad E_{T2\min} = 5 \text{ GeV}. \quad (1)$$

So far this scenario includes infrared sensitive parts of phase space, where  $E_{T1} \simeq E_{T2} \simeq E_{T\min}$ . These phase space regions can be avoided by additional harder cuts on either the  $E_{T1}$  of the hardest jet, on the average  $\overline{E_T}$  of both jets or on the invariant dijet mass  $M_{jj}$ . These different choices are varied in scenario 1(a-c). The central choice will be an additional cut on  $E_{T1} > 8 \text{ GeV}$ . Different values of this cut are tested in scenario 2(a-d).

Starting from the central scenario we also vary the ranges of the kinematical variables  $Q^2$  (scenario 3(a-d)) and  $y$  (scenario 4(a-c)). Further comparisons are dedicated to phase space regions which are irrelevant for experimental analyses, but helpful to test the programs. In scenario 5(a-c) we compare the programs for softer transverse jet energy cuts. Scenario 6(a-c) is

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<sup>1</sup>The official DISENT program does not take into account the running of the electromagnetic coupling constant. We have modified the official DISENT program to include this.

defined by the requirement of a difference in the transverse jet energies. The only contributions to these cross sections come from 3-parton final states in  $\mathcal{O}(\alpha_s^2)$  such that we are left with a leading order prediction.

The various scenarios differ from the central scenario (1) as follows:

SCENARIO 1	
different ways to avoid infrared sensitive regions	
No.	additional jet cut
1 a)	$E_{T1\min} > 8 \text{ GeV}$
1 b)	$M_{jj} > 25 \text{ GeV}$
1 c)	$(E_{T1} + E_{T2}) > 17 \text{ GeV}$

SCENARIO 2	
different $E_{T1\min}$ cuts	
No.	$E_{T1\min}/\text{GeV}$
2 a)	8
2 b)	15
2 c)	25
2 d)	40

SCENARIO 3		
different $Q^2$ ranges		
add. cut $E_{T1\min} = 8 \text{ GeV}$		
No.	$Q_{\min}^2/\text{GeV}^2$	$Q_{\max}^2/\text{GeV}^2$
3 a)	3	4
3 b)	30	40
3 c)	300	400
3 d)	3000	4000

SCENARIO 4		
extreme $y$ regions		
add. cut $E_{T1\min} = 8 \text{ GeV}$		
No.	$y_{\min}$	$y_{\max}$
4 a)	0.01	0.05
4 b)	0.2	0.6
4 c)	0.9	0.95

SCENARIO 5		
probing softer regions		
No.	$E_{T2\min}/\text{GeV}$	$E_{T1\min}/\text{GeV}$
5 a)	1	2
5 b)	2	3
5 c)	3	4

SCENARIO 6	
add. cut on the difference of the jet $E_T$	
No.	$(E_{T1} - E_{T2}) >$
6 a)	1 GeV
6 b)	2 GeV
6 c)	3 GeV

### 3.3 Numerical Comparisons

An overview of the results of all calculations for the 17 different scenarios is given in the tables in the appendix. The leading order results are shown in the last row for each scenario. These values have been calculated to a numerical precision of typically 0.2%. In all cases we see a perfect agreement between the different programs.

The next-to-leading order calculations for the corresponding scenarios have been performed to a numerical precision of typically 0.3%<sup>2</sup>. In most cases we have tested the stability of the JETVIP results w.r.t. the cutoff parameter  $y_{\text{cut}}$ .

<sup>2</sup>For DISASTER++ the precision is often worse since the calculations by DISASTER++ require significantly more CPU time compared to the other programs.

## DISENT and DISASTER++

The programs DISENT and DISASTER++ which are both based on the subtraction method are in very good overall agreement. In 12 cases their NLO results are in agreement within the quoted errors of typically 0.3%. Only in 5 comparisons (1b, 3c, 3d, 4a, 5a) deviations are seen in the range of 0.6% to 2.2% with a significance of 1.5 – 3.6 standard deviations but without any systematic trend. If we consider that the errors quoted by the programs may sometimes be underestimated<sup>3</sup> this can still be labeled “good agreement”.

In cases where the precision of the NLO calculation is important, the user should therefore not trust the quoted errors but aim for a higher precision.

## MEPJET

MEPJET NLO predictions were found to agree well with DISASTER++ and DISENT for physical PDFs in Ref. [4] (Table 2 and discussion on p. 14). MEPJET’s NLO results for the extreme phase space regions investigated here are typically 5–8% lower than the NLO results obtained with DISENT and DISASTER++. In three cases deviations of about 10% occur. No obvious correlation between the size of the deviation and the value of the  $K$ -factor exists. The  $K$ -factor varies from 1.3 to 7.0, except for case 2d. Here DISASTER++ and DISENT yield a  $K$ -factor of 1.17 and 1.16, respectively, while MEPJET’s  $K$ -factor is close to 1. MEPJET deviates from DISASTER++ by  $2\sigma$  and from DISENT by  $3\sigma$  in this case.

All MEPJET calculations ran with the default cutoff value of  $s_{\min} = 0.1 \text{ GeV}^2$ . To check for possible cutoff dependences additional runs with smaller  $s_{\min}$  values were done for selected cases (see data table). No  $s_{\min}$  dependence was found. Effects potentially introduced by approximations used for the crossing functions were also investigated and found to be not significant. All LO results agree within the statistical errors. In addition perfect agreement between MEPJET, DISASTER++ and DISENT is seen in scenario 6, which tests the real  $\mathcal{O}(\alpha_s^2)$  corrections. What causes the observed discrepancies in full NLO in the extreme phase space regions probed here is currently unknown.

## JETVIP

As proposed in [5] we have started to perform the NLO calculations for the JETVIP program for a cutoff value  $y_{\text{cut}} = 10^{-3}$ . Although some of these results are in agreement to the DIS-ENT/DISASTER++ values (scenarios 2b, 3d, 4a, 5a-c), in the other 11 cases discrepancies of up to 20% are seen. Therefore we have made extensive studies on the  $y_{\text{cut}}$  dependence of the JETVIP results in the range  $10^{-6} \leq y_{\text{cut}} \leq 10^{-2}$ . Only in scenario 6, where only real corrections of  $\mathcal{O}(\alpha_s^2)$  are tested, the results become stable for  $y_{\text{cut}} \simeq 10^{-4}$ . For all NLO results we observe a significant cutoff dependence. Since the independence on the cutoff is the most important test of the successful implementation of the phase space slicing method the strong  $y_{\text{cut}}$  dependence of the JETVIP results is worrisome.

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<sup>3</sup>For the DISENT program the same cross section has been repeatedly calculated [13]. The statistically independent results were roughly Gaussian distributed in the central region, with a width compatible with the error quoted by DISENT. However, significantly larger tails have been seen. The same is likely to be true for the other programs (no similar checks have been made).

Especially at very small values of  $10^{-6} \leq y_{\text{cut}} \leq 10^{-5}$  no convergence of the results is seen. In scenario 1a we have repeated the calculation at  $y_{\text{cut}} = 10^{-5}$  with fourfold statistics. While the quoted errors are 2.6% and 0.4%, respectively, both results deviate by 15%. This is a clear indication that at these small  $y_{\text{cut}}$  values the quoted errors are not reliable.

At intermediate values  $10^{-4} \leq y_{\text{cut}} \leq 10^{-3}$  large  $y_{\text{cut}}$  dependencies (above 4%) are observed in four scenarios (2c, 2d, 3a, 6a) only. In the other 13 scenarios the dependence is below 3%. In 11 of these cases the JETVIP results at  $y_{\text{cut}} = 10^{-4}$  agree within this level of precision with the DISENT/DISASTER++ results. The other two results 1b, 1c deviate by 10% and 4.5% from the DISENT/DISASTER++ results.

## 4 Summary

We have compared four different programs for NLO calculations of jet cross sections in  $ep$  collisions: DISENT, DISASTER++, JETVIP and MEPJET. Dijet cross sections in different ranges of  $Q^2$ ,  $y$ ,  $E_{T,\text{jet}}$  have been calculated in leading order (LO) and in next-to-leading order (NLO). All calculations are performed to a numerical precision of typically 0.2% (LO) and 0.3% (NLO).

While the leading order predictions of all programs agree within the numerical precision of 0.2%, our comparisons show that in NLO only the calculations of DISENT and DISASTER++ can be said to be in good agreement.

MEPJET shows systematic deviations of being typically 5–8% lower than DISENT and DISASTER++. Only the  $\mathcal{O}(\alpha_s^2)$  tree level cross sections are in perfect agreement.

The JETVIP program shows a significant dependence on the phase space slicing parameter  $y_{\text{cut}}$  which has to be understood. Only at intermediate values of  $y_{\text{cut}} \simeq 10^{-4}$  the dependence is reduced. In the cases where the  $y_{\text{cut}}$  dependence within  $10^{-4} \leq y_{\text{cut}} \leq 10^{-3}$  is smaller than 3% the JETVIP results are often comparable with the DISENT and DISASTER++ results.

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## References

- [1] E. Mirkes and D. Zeppenfeld, Phys. Lett. **B380** (1996) 205.
- [2] E. Mirkes, preprint TTP-97-39 (1997), hep-ph/9711224.
- [3] S. Catani and M.H. Seymour, Nucl. Phys. **B485** (1997) 291.
- [4] D. Graudenz, hep-ph/9710244.
- [5] B. Pötter, Comput. Phys. Commun. **119** (1999) 45.
- [6] T. Hadig and G. McCance, to appear in: Proceedings of the Workshop on Monte Carlo Generators for HERA Physics, eds. G. Grindhammer, G. Ingelman, H. Jung, T. Doyle, DESY (1999).
- [7] H. Plochow-Besch, Comput. Phys. Commun. **75** (1993) 396.
- [8] CTEQ Collab., H.L. Lai et al., Phys. Rev. **D 55** (1997) 1280.
- [9] S. D. Ellis, D. E. Soper, Phys. Rev. **D48** (1993) 3160;  
S. Catani, Yu. L. Dokshitzer, M. H. Seymour and B. R. Webber, Nucl. Phys. **B406** (1993) 187.
- [10] M.H. Seymour, Nucl. Phys. **B 513** (1998) 269.
- [11] M. Wobisch and T. Wengler, to appear in: Proceedings of the Workshop on Monte Carlo Generators for HERA Physics, eds. G. Grindhammer, G. Ingelman, H. Jung, T. Doyle, DESY (1999), hep-ph/9907280.
- [12] J. Huth et al., Proceedings of the Summer Study on High Energy Physics, Snowmass, Colorado (1990) 134.
- [13] M.H. Seymour, private communication.

# A Numerical Results

Here we list all available numerical results. The last line for each scenario contains the leading-order results.

scenario	DISASTER++	DISENT	JETVIP	MEPJET
1 a)	119.82 $\pm$ 0.411	119.54 $\pm$ 0.33	113.42 $\pm$ 0.10 ( $y_{\text{cut}} = 10^{-2}$ ) 121.41 $\pm$ 0.19 ( $y_{\text{cut}} = 10^{-3}$ ) 121.69 $\pm$ 0.77 ( $y_{\text{cut}} = 10^{-4}$ ) 99.6 $\pm$ 2.6 ( $y_{\text{cut}} = 10^{-5}$ ) 114.98 $\pm$ 0.44 ( $y_{\text{cut}} = 10^{-5}$ ) 75.7 $\pm$ 2.7 ( $y_{\text{cut}} = 10^{-6}$ )	113.45 $\pm$ 0.21 ( $s_{\text{min}} = 0.1$ ) 113.3 $\pm$ 3.5 ( $s_{\text{min}} = 0.0001$ )
LO:	41.662 $\pm$ 0.083	41.769 $\pm$ 0.061	41.745 $\pm$ 0.033	41.722 $\pm$ 0.032
1 b)	82.83 $\pm$ 0.44	81.02 $\pm$ 0.49	93.58 $\pm$ 0.22 ( $y_{\text{cut}} = 10^{-3}$ ) 91.11 $\pm$ 0.49 ( $y_{\text{cut}} = 10^{-4}$ ) 83.55 $\pm$ 1.1 ( $y_{\text{cut}} = 10^{-5}$ )	78.55 $\pm$ 0.16
LO:	30.57 $\pm$ 0.07	30.59 $\pm$ 0.05	30.54 $\pm$ 0.05	30.56 $\pm$ 0.01
1 c)	72.26 $\pm$ 0.30	72.05 $\pm$ 0.28	77.15 $\pm$ 0.16 ( $y_{\text{cut}} = 10^{-3}$ ) 75.46 $\pm$ 0.37 ( $y_{\text{cut}} = 10^{-4}$ ) 66.18 $\pm$ 1.36 ( $y_{\text{cut}} = 10^{-5}$ )	67.57 $\pm$ 0.21
LO:	35.155 $\pm$ 0.072	35.172 $\pm$ 0.052	35.184 $\pm$ 0.050	35.141 $\pm$ 0.024

scenario	DISASTER++	DISENT	JETVIP	MEPJET
2 a)	as 1 a)			
2 b)	16.585 $\pm$ 0.092	16.526 $\pm$ 0.051	16.668 $\pm$ 0.031 ( $y_{\text{cut}} = 10^{-3}$ ) 16.302 $\pm$ 0.071 ( $y_{\text{cut}} = 10^{-4}$ ) 13.668 $\pm$ 0.160 ( $y_{\text{cut}} = 10^{-5}$ )	15.743 $\pm$ 0.078
LO:	6.185 $\pm$ 0.020	6.222 $\pm$ 0.011	6.214 $\pm$ 0.005	6.221 $\pm$ 0.003
2 c)	2.0809 $\pm$ 0.0273	2.0519 $\pm$ 0.0080	1.9563 $\pm$ 0.0049 ( $y_{\text{cut}} = 10^{-3}$ ) 1.7987 $\pm$ 0.0119 ( $y_{\text{cut}} = 10^{-4}$ ) 1.0962 $\pm$ 0.0260 ( $y_{\text{cut}} = 10^{-5}$ )	1.9084 $\pm$ 0.0083
LO:	1.0230 $\pm$ 0.0046	1.0221 $\pm$ 0.0022	1.0255 $\pm$ 0.0009	1.0250 $\pm$ 0.0005
2 d)	0.1398 $\pm$ 0.0052	0.1403 $\pm$ 0.0011	0.1124 $\pm$ 0.0007 ( $y_{\text{cut}} = 10^{-3}$ ) 0.0772 $\pm$ 0.0014 ( $y_{\text{cut}} = 10^{-4}$ )	0.1229 $\pm$ 0.0047
LO:	0.1197 $\pm$ 0.0014	0.12125 $\pm$ 0.00036	0.12073 $\pm$ 0.00016	0.12087 $\pm$ 0.000064

scenario	DISASTER++	DISENT	JETVIP	MEPJET
3 a)	341.2 $\pm$ 1.7	339.1 $\pm$ 1.2	315.9 $\pm$ 0.4 ( $y_{\text{cut}} = 10^{-3}$ ) 340.0 $\pm$ 0.7 ( $y_{\text{cut}} = 10^{-4}$ ) 296.6 $\pm$ 2.0 ( $y_{\text{cut}} = 10^{-5}$ )	331.49 $\pm$ 0.42 ( $s_{\text{min}} = 0.1$ ) 334.96 $\pm$ 1.31 ( $s_{\text{min}} = 0.01$ ) 336 $\pm$ 14 ( $s_{\text{min}} = 0.001$ )
LO:	48.418 $\pm$ 0.100	48.423 $\pm$ 0.081	48.363 $\pm$ 0.040	48.397 $\pm$ 0.040
3 b)	as 1 a)			
3 c)	26.848 $\pm$ 0.061	26.680 $\pm$ 0.051	26.259 $\pm$ 0.139 ( $y_{\text{cut}} = 10^{-3}$ ) 26.79 $\pm$ 0.094 ( $y_{\text{cut}} = 10^{-4}$ ) 23.894 $\pm$ 0.407 ( $y_{\text{cut}} = 10^{-5}$ )	24.684 $\pm$ 0.050
LO:	16.938 $\pm$ 0.022	16.936 $\pm$ 0.016	16.928 $\pm$ 0.008	16.918 $\pm$ 0.011
3 d)	1.9975 $\pm$ 0.0033	1.9852 $\pm$ 0.0029	1.9657 $\pm$ 0.0061 ( $y_{\text{cut}} = 10^{-3}$ ) 1.9946 $\pm$ 0.0066 ( $y_{\text{cut}} = 10^{-4}$ ) 1.7194 $\pm$ 0.0179 ( $y_{\text{cut}} = 10^{-5}$ )	1.8917 $\pm$ 0.0038
LO:	1.4982 $\pm$ 0.0017	1.4967 $\pm$ 0.0013	1.4956 $\pm$ 0.0013	1.4966 $\pm$ 0.0010



scenario	DISASTER++	DISENT	JETVIP	MEPJET
4 a)	19.218 $\pm$ 0.143	18.959 $\pm$ 0.068	18.818 $\pm$ 0.051 ( $y_{\text{cut}} = 10^{-3}$ ) 18.470 $\pm$ 0.041 ( $y_{\text{cut}} = 10^{-4}$ ) 18.896 $\pm$ 0.167 ( $y_{\text{cut}} = 10^{-5}$ )	17.190 $\pm$ 0.037
LO:	11.611 $\pm$ 0.038	11.573 $\pm$ 0.022	11.590 $\pm$ 0.010	11.587 $\pm$ 0.006
4 b)	as 1 a)			
4 c)	6.424 $\pm$ 0.027	6.448 $\pm$ 0.018	6.299 $\pm$ 0.059 ( $y_{\text{cut}} = 10^{-3}$ ) 6.356 $\pm$ 0.040 ( $y_{\text{cut}} = 10^{-4}$ ) 6.243 $\pm$ 0.155 ( $y_{\text{cut}} = 10^{-5}$ )	6.086 $\pm$ 0.028
LO:	2.1612 $\pm$ 0.0058	2.1615 $\pm$ 0.0031	2.173 $\pm$ 0.013	2.1598 $\pm$ 0.0015

scenario	DISASTER++	DISENT	JETVIP	MEPJET
5 a)	1676.2 $\pm$ 4.0	1655.6 $\pm$ 4.2	1654.3 $\pm$ 6.0 ( $y_{\text{cut}} = 10^{-3}$ ) 1678.6 $\pm$ 20.4 ( $y_{\text{cut}} = 10^{-4}$ )	1489.8 $\pm$ 3.2
LO:	845.40 $\pm$ 1.04	844.71 $\pm$ 0.70	844.84 $\pm$ 0.83	844.67 $\pm$ 0.45
5 b)	973.8 $\pm$ 2.6	970.1 $\pm$ 2.4	970.3 $\pm$ 3.0 ( $y_{\text{cut}} = 10^{-3}$ ) 989.4 $\pm$ 8.3 ( $y_{\text{cut}} = 10^{-4}$ )	885.9 $\pm$ 2.0
LO:	436.43 $\pm$ 0.62	436.25 $\pm$ 0.43	436.85 $\pm$ 0.68	436.27 $\pm$ 0.23
5 c)	564.5 $\pm$ 1.6	561.9 $\pm$ 1.5	565.6 $\pm$ 1.5 ( $y_{\text{cut}} = 10^{-3}$ ) 573.6 $\pm$ 4.8 ( $y_{\text{cut}} = 10^{-4}$ )	518.0 $\pm$ 0.8
LO:	242.20 $\pm$ 0.37	242.60 $\pm$ 0.28	243.25 $\pm$ 0.36	242.47 $\pm$ 0.23

scenario	DISASTER++	DISENT	JETVIP	MEPJET
6 a)	126.24 $\pm$ 0.44	126.92 $\pm$ 0.47	118.13 $\pm$ 0.05 ( $y_{\text{cut}} = 10^{-3}$ ) 122.94 $\pm$ 0.05 ( $y_{\text{cut}} = 10^{-4}$ ) 123.01 $\pm$ 0.05 ( $y_{\text{cut}} = 10^{-5}$ ) 123.23 $\pm$ 0.22 ( $y_{\text{cut}} = 10^{-6}$ )	126.08 $\pm$ 0.20
6 b)	56.30 $\pm$ 0.26	56.02 $\pm$ 0.25	54.78 $\pm$ 0.02 ( $y_{\text{cut}} = 10^{-3}$ ) 55.30 $\pm$ 0.03 ( $y_{\text{cut}} = 10^{-4}$ ) 55.30 $\pm$ 0.03 ( $y_{\text{cut}} = 10^{-5}$ ) 55.30 $\pm$ 0.03 ( $y_{\text{cut}} = 10^{-6}$ )	55.90 $\pm$ 0.10
6 c)	27.15 $\pm$ 0.16	27.13 $\pm$ 0.07	26.94 $\pm$ 0.01 ( $y_{\text{cut}} = 10^{-3}$ ) 27.00 $\pm$ 0.02 ( $y_{\text{cut}} = 10^{-4}$ ) 27.00 $\pm$ 0.02 ( $y_{\text{cut}} = 10^{-5}$ ) 27.01 $\pm$ 0.02 ( $y_{\text{cut}} = 10^{-6}$ )	27.14 $\pm$ 0.05